Quarkonium Formation Time in a Model-Independent Approach

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Abstract

We use dispersion relations to reconstruct, in a model–independent way, the formation dynamics of heavy quarkonium from the experimental data on $e^+e^- \to \bar{Q}Q$ annihilation. We extract a distribution of formation times with a mean value for the J/ψ, $<\tau_{J/\psi}>= 0.44$ fm; and for the Υ, $<\tau_{\Upsilon}>= 0.32$ fm. The corresponding widths of these distributions are given by $\Delta\tau_{J/\psi} = 0.31$ fm and $\Delta\tau_{\Upsilon} = 0.28$ fm. This information can be used as an input in modeling of heavy quarkonium production on nuclear targets.

The creation of a heavy quark-antiquark pair occurs at small distances ($\sim 1/m_Q$) and produces compact $\bar{Q}Q$ states which later transform into physical heavy hadrons. In the case of quarkonium production on nuclear targets, this evolution can cause observable effects [1]. While these "formation" [2] effects can in principle be evaluated in Glauber–Gribov theory [3], in practice this calculation is difficult to perform in a model–independent way, since it requires the knowledge of all off-diagonal components of the quarkonium–nucleon scattering amplitude. Therefore one often uses a simplistic approach, in which the evolution of the quark-antiquark pair is mimicked by a fixed "formation time", during which the interactions of the pair are different from the interactions of the physical quarkonium; depending on the color state of the pair, the interactions of the "unformed" pair can be either suppressed ("color transparency" [1]) or enhanced, if the pair are formed in the color octet state [4].

In the literature one can find different prescriptions for the formation time τ_f. A still popular viewpoint, for instance, is to assume a universal parameter on the order of some characteristic hadronic scale, say, $\tau_f \sim m_\rho^{-1}$. Alternatively, one considers the classical expansion of the heavy quark pair and defines formation time as the time when the separation of the pair reaches the size of a physical quarkonium state [5]. Much work has been done also on the quantum-mechanical approach to quarkonium formation, where the expansion of a small initial wave packet is controlled by the spacings of the bound state mass spectrum [6],[7]. In this work we address the problem of formation time starting from the idea that all essential information about the expansion of the wave packet is contained in the correlator of the hard scattering operator [8].

Let us consider the space-time correlator of an operator $\hat{J}$, which produces from initial state $\mid i >$ the $(\bar{Q}Q)$ state with certain quantum numbers

$$\Pi(x) = \left\langle i \mid T\{\hat{J}(x)\hat{J}(0)\} \mid i \right\rangle \tag{1}$$

The basic expression which allows the use of experimental data for extracting information from this correlator is the dispersion relation, which in coordinate representation [9], [10] takes the form:

$$\Pi(x) = \frac{1}{\pi} \int Im\Pi(s) D\left(\sqrt{s}, x^2\right) ds, \tag{2}$$

where

$$D\left(\sqrt{s}, \tau^2 = -x^2\right) = \frac{\sqrt{s}}{4\pi^2 \tau} K_1\left(\sqrt{s}\tau\right) \tag{3}$$

is the relativistic causal propagator in the coordinate representation; K_1 is the Hankel function. The expression (2) relates the behavior of the correlator (1) to experimentally measurable cross sections for physical processes. For example, in the case of $(\bar{Q}Q)$ pair production in e^+e^- annihilation one has simply

$$Im\Pi(s) = \frac{s}{(4\pi\alpha)^2}\, \sigma(e^+e^- \to \bar{Q}Q; s). \tag{4}$$

The physical meaning of eq.(2) is transparent: it represents the correlator as a superposition of propagators of physical states, each with the weight proportional to the probability of their production in a hard process.

Thus it is possible to extract information about the space-time evolution of various states in a given hadronic channel with fixed quantum numbers directly from experimental data. For example, we can define the formation time of the ground state with mass m by the time τ_f at which the correlator approaches its asymptotic behavior

$$\Pi(\tau) \sim \tau^{-3/2}\ exp(-m\tau); \tag{5}$$

note that $\tau = it$ is Euclidian time.

To illustrate the notion of formation time in more detail, let us use the following simple example – assume that the spectral density (4) consists of two narrow states of identical strength, i.e.

$$Im\Pi(s) \sim \delta(s - m_1^2) + \delta(s - m_2^2). \tag{6}$$

At large time τ, the correlator (1) will look like

$$\Pi(\tau) \sim \tau^{-3/2}\ exp(-m_1\tau)\left[m_1^{1/2} + m_2^{1/2}\ exp(-(m_2 - m_1)\tau)\right]. \tag{7}$$

The use of criterion (5) therefore leads to

$$\tau_f \sim \frac{1}{m_2 - m_1}. \tag{8}$$

Let us now introduce invariant $\lambda \equiv t^2 - r^2$ and decompose (3) in the following way [9]:

$$D\left(\sqrt{s}, \lambda\right) = \frac{1}{4\pi}\ \delta(\sqrt{\lambda}) - \frac{\sqrt{s}}{8\pi\sqrt{\lambda}}\ \theta(\lambda)\left[J_1(\sqrt{s}\sqrt{\lambda}) - iN_1(\sqrt{s}\sqrt{\lambda})\right] + \tag{9}$$
$$+\frac{i\sqrt{s}}{4\pi^2\sqrt{-\lambda}}\ \theta(-\lambda)K_1(\sqrt{s}\sqrt{-\lambda});$$

It is clear from (9) that the formation time τ_f extracted from the Euclidian asymptotics (5) of the correlator (2) will also determine the propagation of the quark-antiquark state in Minkowski space, with $\lambda > 0$. Let us introduce the light-cone variables $x^+ = t + z$, $x^- = t - z$, so that $\lambda = t^2 - r^2 = x^+x^-$, and conjugate momenta $p^- = p_0 - p_z \simeq m^2/2p_z$ and $p^+ = p_0 + p_z$; as can be seen from (2), (6) and (9), in Minkowski space the criterion (5) leads to

$$l_f = \Delta x^+ \sim \frac{1}{\Delta p^-} \simeq \frac{2p}{m_2^2 - m_1^2}, \tag{10}$$

i.e. the formation length of quarkonium l_f grows linearly with its momentum $p = p_z$ in the lab frame. The formation time (8) extracted from Euclidian asymptotics (5) of the correlator is related to the formation length (10) by the Lorentz transformation:

$$l_f \simeq \frac{p}{(m_1 + m_2)/2}\ \tau_f = \frac{2p}{m_2^2 - m_1^2}, \tag{11}$$

where $(m_1+m_2)/2$ can be interpreted as a characteristic mass of the wave packet. One can recognize this length also as the inverse of the longitudinal momentum transfer [3]

$$\Delta q_{\|} \simeq \sqrt{p_0^2 - m_1^2} - \sqrt{p_0^2 - m_2^2} \simeq \frac{m_2^2 - m_1^2}{2p}, \tag{12}$$

which accompanies the transition between the two hadronic states at high energy. The wave packet can escape from a nucleus of radius R_A before being "formed" if $l_f \gg R_A$ or, equivalently, if

$$\frac{m_2^2 - m_1^2}{2p} R_A \ll 1. \tag{13}$$

This condition was derived many years ago by Gribov [3].

One can clearly see that the value of formation time is by no means a universal constant. It depends both on the properties of the interaction $\hat{J}$ and on the entire spectrum of states in a given hadronic channel. Models using some universal value of formation time are therefore misleading. Let us note that eqs.(1-4) indicate that the space-time picture of a hard process can be equivalently described in the language of the spectrum of hadronic excitations which are formed as a result of this hard process. If the operator J is of a short-range nature, then it produces a compact object; Eq.(2) tells us that it is composed of many normal-sized hadronic states.

Since there exist high quality data on the production of heavy quark states in e^+e^- annihilation, we will concentrate in the remainder of this Letter on the formation dynamics of vector $J^{PC} = 1^{--}$ quarkonium states. In this case the Fourier transform of the correlator $\Pi_{\mu\nu} = \langle 0|T\{J_\mu(x)J_\nu(0)\}|0\rangle$ has the following familiar structure:

$$i \int d^4x\ e^{iqx}\ \Pi_{\mu\nu}(x) = \Pi(q^2)(q_\mu q_\nu - q^2 g_{\mu\nu}). \tag{14}$$

We use the cross section for $e^+e^- \to \mu^+\mu^-$ annihilation,

$$\sigma(e^+e^- \to \mu^+\mu^-; s) = \frac{4\pi\alpha^2}{3s}, \tag{15}$$

to express (4) in terms of the familiar ratio

$$R(s) = \frac{\sigma(e^+e^- \to \bar{Q}Q; s)}{\sigma(e^+e^- \to \mu^+\mu^-; s)}, \tag{16}$$

resulting in

$$Im\Pi(s) = \frac{1}{12\pi} R(s). \tag{17}$$

Contracting Lorentz indices and using the relation $(q_\mu q_\nu - q^2 g_{\mu\nu})|_{\mu=\nu} = 3q^2 = -3\partial^2$, one can write down the expression (2) as [9], [10]

$$\Pi(x) = -3\frac{1}{12\pi^2} \int_0^\infty R(s)\ \partial^2 D(\sqrt{s}, x^2) ds. \tag{18}$$

Since the scalar propagator satisfies the equation

$$(-\partial^2 - s)D(\sqrt{s}, x^2) = -\delta(x), \tag{19}$$

at $x^2 \neq 0$ the final result takes the following form:

$$\Pi(x) = \frac{1}{4\pi^2}\int_0^\infty sR(s)D(\sqrt{s}, x^2)ds. \tag{20}$$

We parametrize the ratio $R(s)$ in terms of narrow resonances plus a continuum contribution:

$$R(s) = \sum_i R_i(s) + R_{cont}(s), \tag{21}$$

where

$$R_i(s) = (2J_i + 1)\frac{3}{4\alpha^2}\frac{B(\Psi_i \to e^+e^-)\Gamma_i^2}{(\sqrt{s} - M_i)^2 + \frac{\Gamma_i^2}{4}} \tag{22}$$

is the contribution of the Ψ_i resonance with spin J_i, mass M_i and total width Γ_i. The continuum contribution can be described with a reasonable accuracy as

$$R_{cont}(s) = 3e_Q^2\Theta(s - s_{th}), \tag{23}$$

where e_Q is the electric charge of the heavy quark.

¿From (3), (20), and (22) we calculate the contribution of each narrow resonance term

$$\Pi_i(\tau) = \frac{3\sigma_i\Gamma_i M_i^6}{64\pi^4\alpha^2\tau}K_1(M_i\tau), \tag{24}$$

where

$$\sigma_i = \frac{4\pi}{{M_i}^2}(2J_i + 1)B(\Psi_i \to e^+e^-) \tag{25}$$

is the magnitude of the annihilation cross section at the resonance peak $\sqrt{s} = M_i$.

The continuum contribution is

$$\Pi_{cont}(\tau) = \frac{3{e_Q}^2}{8\pi^4\tau^6}\int_{2M_{th}\tau}^\infty x^4 K_1(x)dx, \tag{26}$$

where the open flavor threshold $s_{th} = 4{M_{th}}^2$.

The standard limiting forms of the Bessel function then give the behavior of these terms for large and small Euclidean times:

$$\Pi_i(\tau \to \infty) = \frac{3\sigma_i\Gamma_i {M_i}^{\frac{11}{2}}}{64\pi^3\alpha^2\sqrt{2\pi}\tau^{\frac{3}{2}}}e^{-M_i\tau} \tag{27}$$

$$\Pi_{cont}(\tau \to \infty) = \frac{3{e_Q}^2 {M_{th}}^{\frac{7}{2}}}{\pi^{\frac{7}{2}}\tau^{\frac{5}{2}}}e^{-2M_{th}\tau} \tag{28}$$

$$\Pi_i(\tau \to 0) = \frac{3\sigma_i \Gamma_i {M_i}^5}{64\pi^4 \alpha^2 \tau^2} \tag{29}$$

$$\Pi_{cont}(\tau \to 0) = \frac{6{e_Q}^2}{\pi^4 \tau^6} \tag{30}$$

One sees that as $\tau \to \infty$, the correlator is dominated by the ground state contribution, but as $\tau \to 0$, the continuum contribution always dominates, independent of the mass spectrum.

We now consider the fraction F_i of a given state Ψ_i present in the $\bar{Q}Q$ correlator at a given Euclidian time τ,

$$F_i(\tau) = \frac{\Pi_i(\tau)}{\Pi(\tau)}, \tag{31}$$

By definition, $F_i(\tau) \leq 1$, and for the ground state

$$F_0(\tau \to \infty) = 1. \tag{32}$$

The formation time τ_f for the ground state can be defined now by the time at which the correlator is dominated by the ground state contribution, or equivalently when the relation (32) is satisfied with a certain accuracy.

To illustrate the time evolution of the contents of the $\bar{Q}Q$ correlator, we plot in Fig. 1 the functions $F_i(\tau)$ for J/Ψ and for Υ. For the charm case, we have included the J/Ψ and Ψ' resonances, the continuum starting at $M_{th} = M_D$, and also the next three prominent $\Psi(nS)$ resonances above open charm threshold. For the bottom quark case, we have included the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resonances, the continuum starting at $M_{th} = M_B$, and again three prominent S-wave resonances above threshold. To exhibit the effect of the continuum contribution for small τ, we also show curves for these ratios with the continuum omitted.

Also shown in Fig. 1 are the discrete values of formation time $\tau_f^{(o)}$ which result from the simple estimates in Eq. 8, using the inverse mass spacing between the ground and first excited states. One sees that the rapid increase in the $F_i(\tau)$ ratios occurs generally in the region of $\tau_f^{(o)}$ values, but of course a range of formation time values is involved in the approach towards the ground state dominance of the correlator. Operationally, we can then interpret the ratio functions $F_i(\tau)$ as the generalization of the form

$$F_i^{(o)}(\tau) \equiv \theta(\tau - \tau_f^{(o)}) \tag{33}$$

which would be expected if the narrow resonance state Ψ_i had been absent until its instantaneous formation at the time $\tau_f^{(o)}$. We are then led to interpret the derivative of the $F_i(\tau)$ ratios as a continuous distribution $\mathcal{P}(\tau)$ of formation times which occur in the evolution of the real correlator,

$$\mathcal{P}_i(\tau) = \frac{dF_i(\tau)}{d\tau}. \tag{34}$$

These distributions are shown in Fig. 2, again for the ground states of charmonium and bottomonium, and compared with the δ - function position at the corresponding $\tau_f^{(o)}$ values. One sees that the distributions peak at τ somewhat less than these single formation time values. The mean values for these distributions are comparable to the $\tau_f^{(o)}$, but the widths of the distributions are also comparable to the mean. Thus in evaluating any physical quantity which depends on a resonance formation time, one should average this quantity over the distribution in Eq. 34.

In principle, information is also contained in the correlator about the time evolution and formation of the excited states of quarkonium. However, it is not straightforward how to extract this information, since the procedure used for the ground states depends on their dominance at large Euclidean times. We have attempted an approximate procedure, which involves omitting in the dispersion integral all states with lower mass than the state under consideration. This of course neglects the effects of interference in the wave function between the states under consideration and all lower states in the wave packet evolution picture, or equivalently neglects all initial fluctuations with energy less than the mass of the excited state in the virtual state picture. The results for ψ', $\Upsilon(2S)$, and $\Upsilon(3S)$ are shown in Fig. 3. The general shape and parameters are in accord with those for the ground states. It is interesting to note that the relatively longer formation time for the ψ' is a direct result of its closeness to the continuum threshold, whereas one might alternatively attribute this to the larger size of the final physical state. Of course, both of these pictures must be consistent with the same confinement dynamics, and hence may be expected to be related.

In summary, we have shown how the experimental data on the current correlators can be converted to the formation time distributions of the physical states. The necessary information in the case of vector heavy quarkonia is provided by the data on e^+e^- annihilation. The formation time distributions are nonzero in the region of time expected from simple arguments involving the inverse bound state spacings, but in addition show interesting shapes which persist to large times. Our results could be used as an input in phenomenological modeling of quarkonium production in $p - A$ and $A - B$ collisions.

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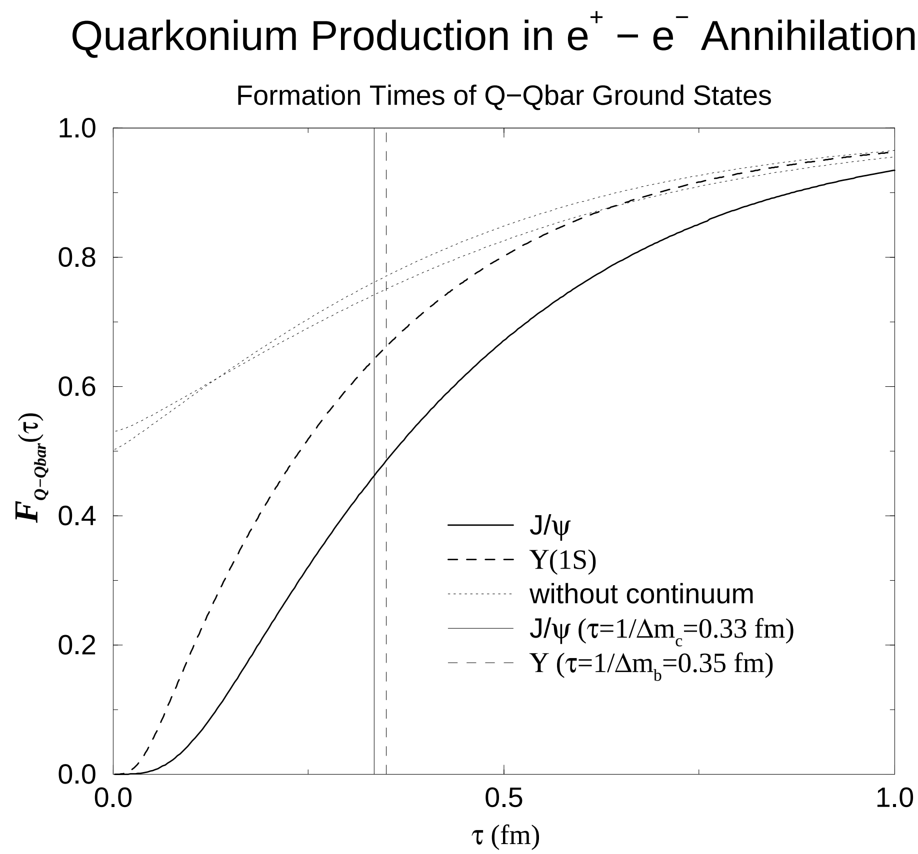

Figure 1: Formation times for the ground states of quarkonium in $e^+ - e^-$ annihilation

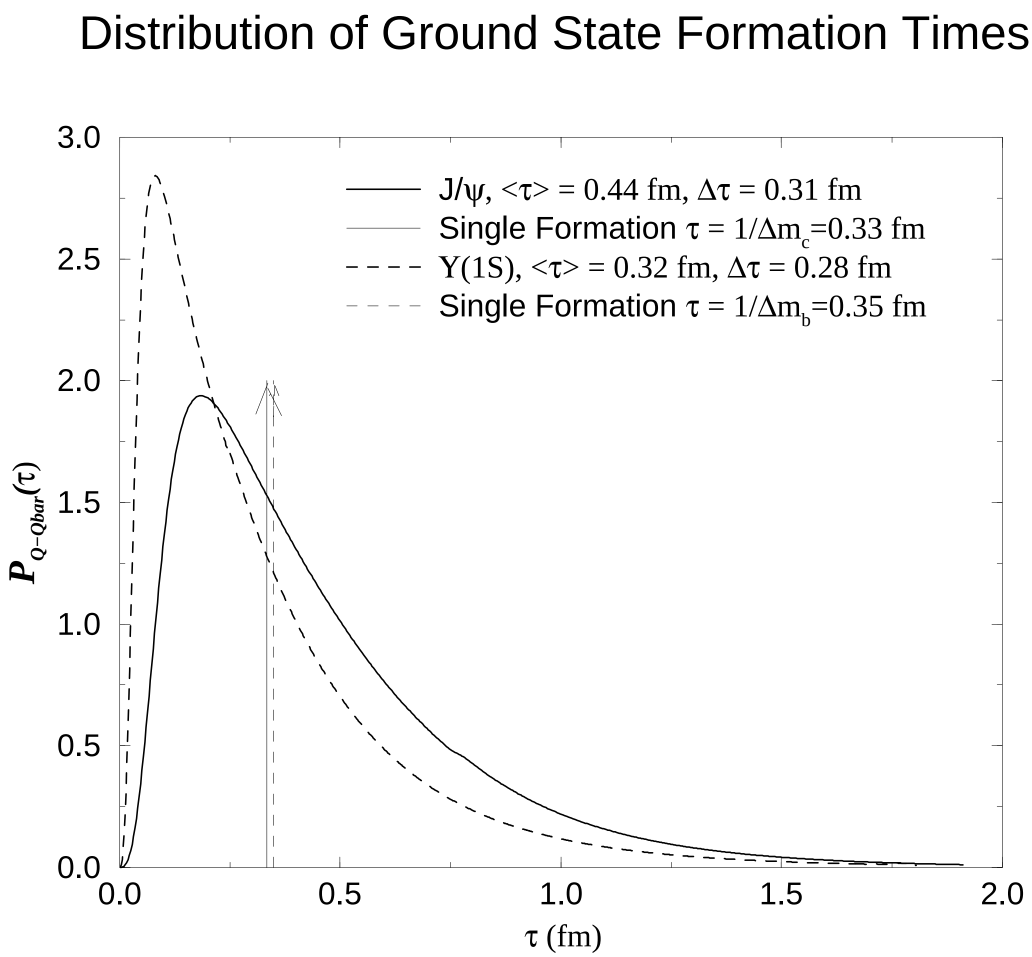

Figure 2: Normalized distribution functions for formation times of J/ψ and Υ in $e^+ - e^-$ annihilation

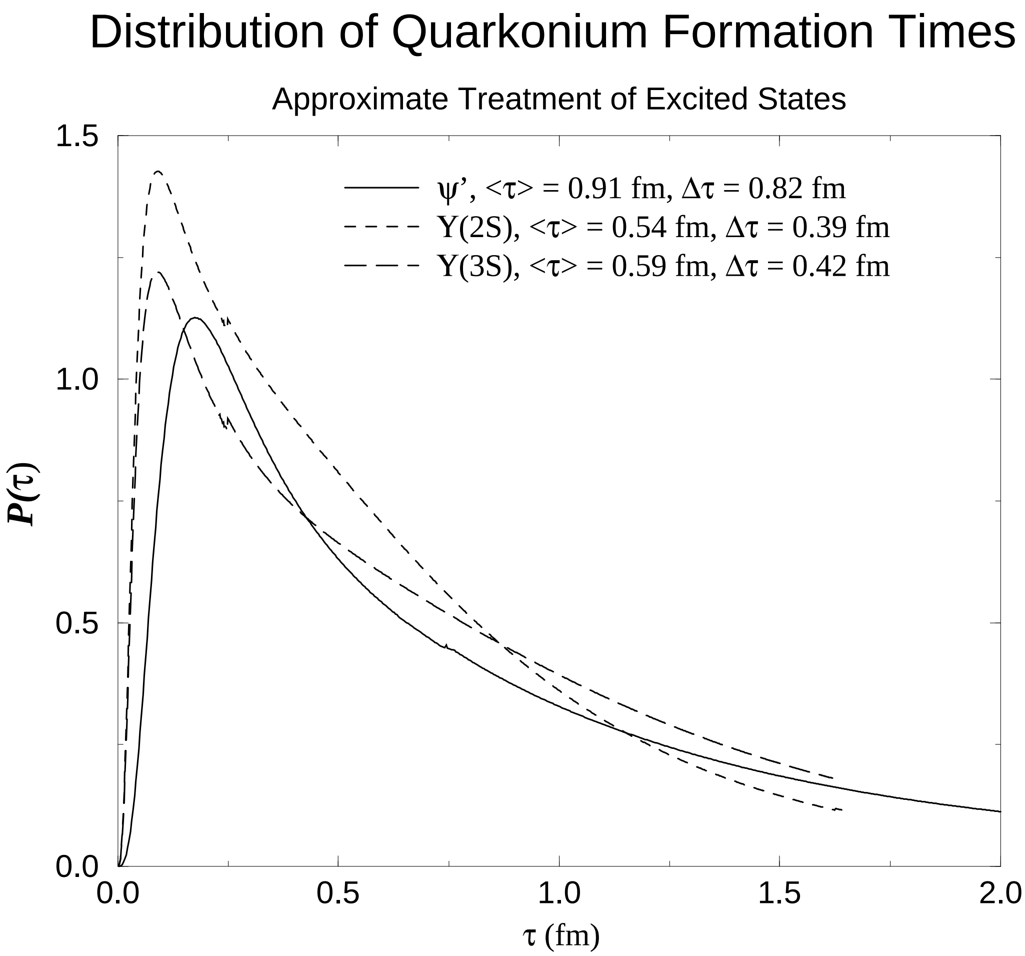

Figure 3: Normalized distribution functions from approximate treatment of formation times for excited quarkonium states in $e^+ - e^-$ annihilation